Confidence Interval of a Ratio of Two Means

Another common situation is that the variable of interest is a quotient (Q) determined by dividing one measurement by another (Q = A/B). A and B are Gaussian variables, and you wish to know the 95% CI for the mean quotient Q. Since A and B can be measured in different units, there is no way to directly combine the SEs (which may also be measured in different units). Instead it is possible to combine the ratios of the SE of A or B divided by the means. If the SE of the denominator B is small compared to B, then you can approximate the CI of the quotient using Equation 35.3:

$$SE_Q = Q\sqrt{\frac{SEM_A^2}{A^2} + \frac{SEM_B^2}{B^2}} \tag{35.3}$$

$$95\%\ CI{:}\ Q - t^* \cdot SE_Q \text{ to } Q + t^* \cdot SE_Q.$$

Since SEM_A is expressed in the same units as A, the ratio is unitless. The ratios of SEM_B/B and SE_Q/Q are also unitless.

If the SEM of the denominator is not small compared to the denominator, then you need to use a more complicated equation, as the CI of the quotient is not symmetrical around the quotient. The derivation of the equations (derived by Fieller) is not intuitive and will not be presented here.

First calculate the intermediate variable g using Equation 35.4:

$$g = \left(t^* \cdot \frac{SEM_B}{B}\right)^2. \tag{35.4}$$

If g has a value greater than or equal to 1, you cannot calculate the CI of the quotient. The value will only be this large when the CI for the denominator (B) includes 0. If the CI of the denominator includes 0, then it is impossible to calculate the CI of the quotient. If g has a value less than 1.0, calculate the CI for the quotient from Equation 35.5:

$$SE_Q = \frac{Q}{(1-g)} \cdot \sqrt{(1-g)\frac{SEM_A^2}{A^2} + \frac{SEM_B^2}{B^2}}. \tag{35.5}$$

$$95\%\ CI{:}\ \frac{Q}{(1-g)} - t^* \cdot SE_Q \text{ to } \frac{Q}{(1-g)} + t^* \cdot SE_Q.$$

Calculate g using Equation 35.4 and look up t* for 95% confidence with the degrees of freedom equal to the total number of values in numerator and denominator minus two.

If the SEM of B is small, then g is very small, the CI is nearly symmetrical around A/B and Equation 35.3 is a reasonable approximation. If the SEM of B is moderately large, then the value of g will be a large fraction, the CI will not be centered around Q and you need Equation 35.5. If the SEM of B is large, then g will be greater than 1.0 and the CI cannot be calculated.

These equations are based on the assumption that the two variables A and B are not paired or matched, and are distributed in a Gaussian manner. If the two variables are paired (and thus correlated) then more complicated equations are needed that also include the correlation coefficient between the two variables.

From Motulsky, Intuitive Biostatistics, page 285-286